Epistemology and philosophy of science 2023/24

Module II: quantum mechanics

Lesson 2

- Quantum mechanics and the measurement problem
  - Wave particle duality: the two-slit experiment

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I. The standard interpretation of the wave function

- In quantum mechanics (QM), the state of a system is mathematically represented by the state vector $|\psi\rangle$
- $|\psi\rangle$ is a vector defined in a complex-valued vector space: the Hilbert space
- Given the state vector, we can define the wave function of a system $\psi(x)$
- The state of a quantum system can thus be equivalently represented by the wave function
- From the mathematical point of view, the wave function is the projection of the state vector in the position basis (i.e. it is the projection of the state vector into an infinite set of position eigenstates):

$$\boldsymbol{\psi(x) = \langle x|\psi\rangle}$$

Standard interpretation of the wave function: the statistical interpretation

In the standard interpretation of QM, the physical meaning of the wave function is linked to $|\psi(x)|^2$. The absolute square of the wave function $|\boldsymbol{\psi(x)}|^{\mathbf{2}}$ indicates the **probability density** to find the particles composing the system if we perform a position measurement on the system. Correspondingly, the integral of the probability density P=$\int|\psi(x)|^2\,dx$ represents the probability to find the particles composing the system in the region defined by the integral if we perform a position measurement on the system. This is known as the Born's statistical interpretation of the wave function.

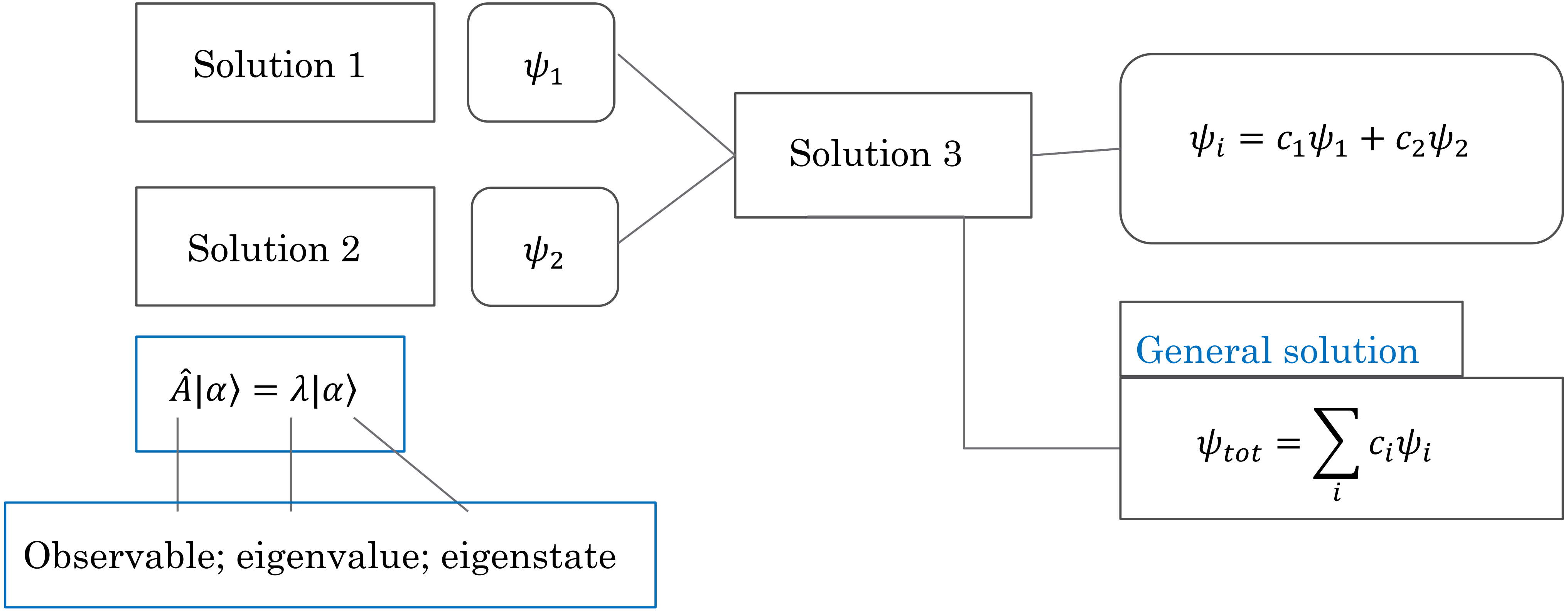
- $|\psi(x)|^2 = \psi(x)^*\psi(x)$ probability density – real-valued function
- P=$\int|\psi(x)|^2\,dx$ probability to find the particle in a given region $\int dx = \Delta x$

II. Dynamics and superposition principle

The wave function evolves according to the Schrödinger equation

$$i\hbar \frac{d}{dt}\psi(x,t) = \hat{H}\,\psi(x,t)$$

The Schrödinger equation is linear:



The measurement problem

1. The Schrödinger equation is linear

2. A measurement is an interaction between micro and macroscopic system

3. The final state of a measurement is a linear superposition of (classically) incompatible apparatus states, e.g. a pointer pointing in opposite directions.

When the system is entangled with a measurement apparatus or a macroscopic object, the quantum superposition represents a macroscopic superposition of classically incompatible states (i.e. a cat dead and alive at the same time)

$$\left(\sum_i c_i\,|\varphi_i(0)\rangle\,\right)\otimes|A(0)\rangle \xrightarrow{Schrödinger} |\Psi(t)\rangle = \sum_i c_i|\varphi_i\rangle|A_i\rangle$$

Linear superposition of different macroscopic states (pointer states) $|A_i\rangle$

Entangled state: system and apparatus are “entangled” in the same state vector $|\Psi(t)\rangle$

The collapse postulate

- A pragmatic solution is given by the ***projection postulate*** *or* ***collapse postulate***

When a "measurement" is performed, the system (superposition of eigenstates) "collapses" instantaneously into one of the eigenstates of the measured observable, with the probability given by the Born rule.

$$|\psi\rangle = \sum_i c_i\, |\alpha_i\rangle \xrightarrow{measurement} |\alpha_j\rangle \; with\; P = \left|c_j\right|^2 = \left|\langle\alpha_j|\psi\rangle\right|^2$$

$$|\Psi(t)\rangle = \sum_i c_i |\varphi_i\rangle |A_i\rangle \xrightarrow{A-measurement} |\varphi_j\rangle |A_j\rangle \text{ with } P = \left|c_j\right|^2$$

After the collapse, the result of the experiment will be a macroscopic apparatus in a well-definite state $|A_j\rangle$ and a quantum system in a well-defined eigenstate of the measured observable $|\varphi_j\rangle$.

Note that in the final state system and apparatus are not entangled anymore

Is the measurement problem really solved?

The collapse postulate is not considered satisfactory by many authors

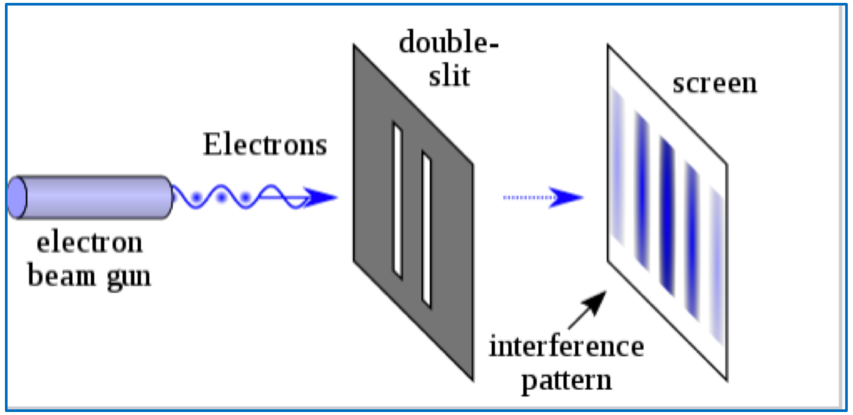
- J. Bell: "Against Measurement" (Speakable and Unspeakable in QM, 1987)
- What physical interactions are "measurements" ?
- A fundamental theory should not be based on the notion of measurement, the measurement is a physical interaction as any other.
- In a "serious" quantum theory, measurement has no special role

a. **Everett theory--Many Worlds Theory** (only linear evolution, many branches). QM without collapse: interactions form a continuous branching between different worlds.

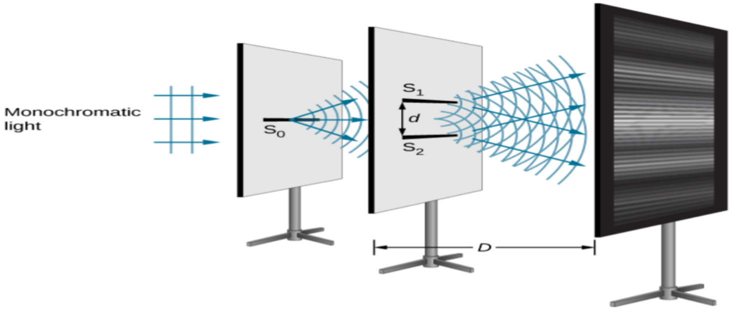
b. **De Broglie-Bohm theory** (wave function *and* particles). There is no collapse, the particles select only one branch/measurement result out of the total superposition.

c. **GRW theory–-spontaneous collapse models**. Modified Schrödinger equation. The wave function does not evolve linearly, but collapse instantaneously at random times.

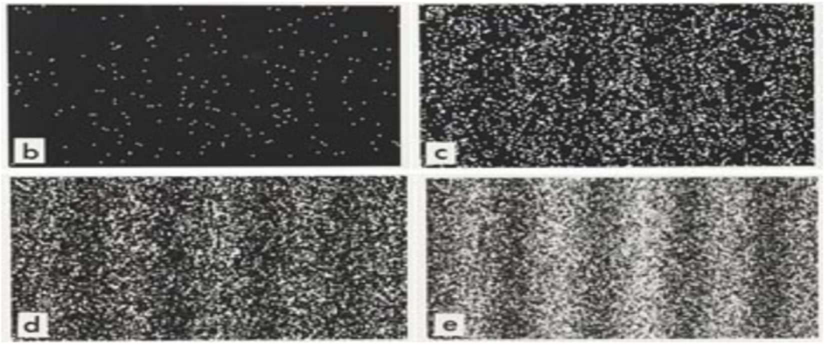
double-slit experiment: wave-particle duality



A beam of electrons pass through a first screen with two slits and impinges on a final screen where electrons are absorbed.

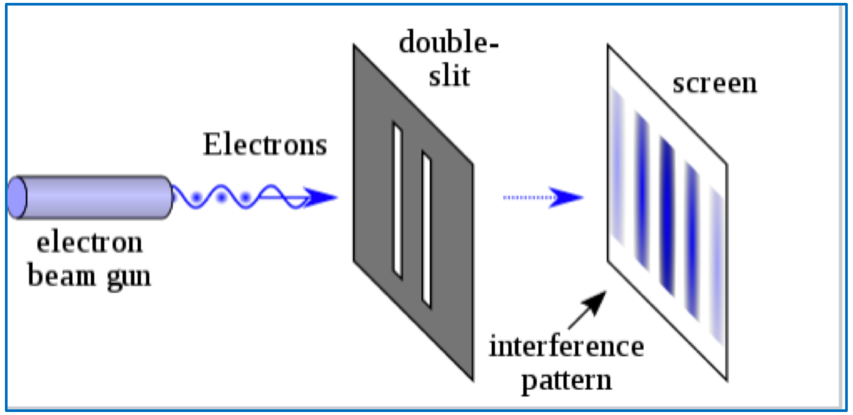


The electron is a wave: constructive and destructive interferences explain the interference patter on the final screen.



The electron is a particle: each electron arrives at the final screen in a discrete unit and in a specific position, like a particle (not like a wave)

double-slit experiment: solution by formalism



A beam of electrons pass through a first screen with two slits and impinges on a final screen where electrons are absorbed.

$$\psi_i(0,0) \xrightarrow{Schrödinger} \psi_1(x,t) + \psi_2(x,t)$$

Probability distribution (final screen)

$$|\psi_1 + \psi_2|^2 = |\psi_1|^2 + |\psi_2|^2 + 2Re[\psi_1^* \psi_2]$$

Interference pattern

The interference pattern is explained by the interference of the two components of the wave function after the slits

- **Quantum superpositions are real even for a single electron**
- **Wave behavior "on flight"**

The wave function of a single electron collapses on the final screen into an eigenstate of the positions

- **Particle behavior in interaction with a macroscopic object (projection postulate)**

References

The measurement problem

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Interpretations of quantum mechanics

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- G. C. Ghirardi, A. Rimini & T. Weber (1986), *Unified Dynamics for Microscopic and Macroscopic Systems*, Physical Review D 34.
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